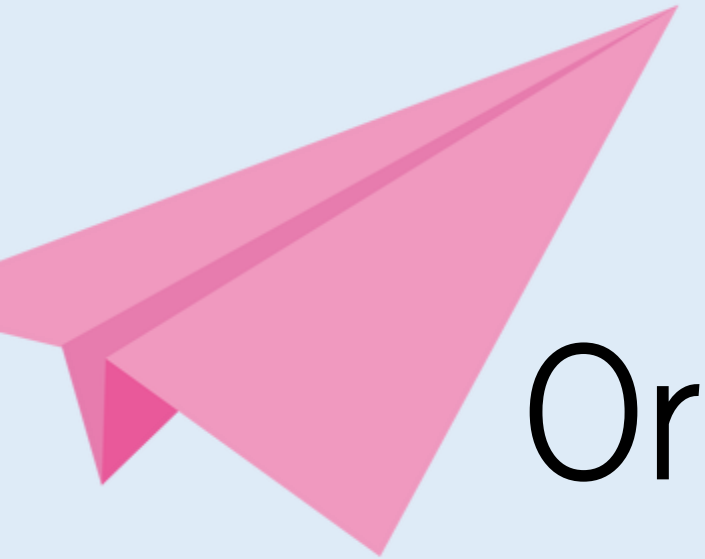


Origami and Regular Polygons

Hyogo Prefecture, Kakogawa Higashi High School
Science & Math Course　Science Research Group1

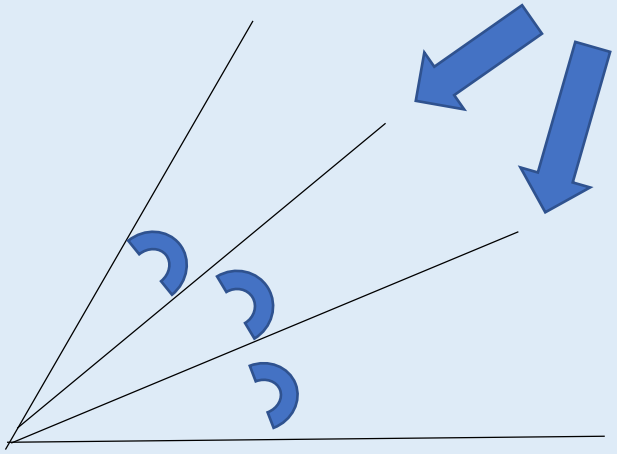
What is Origami?

Origami is the traditional Japanese art form of paper folding.

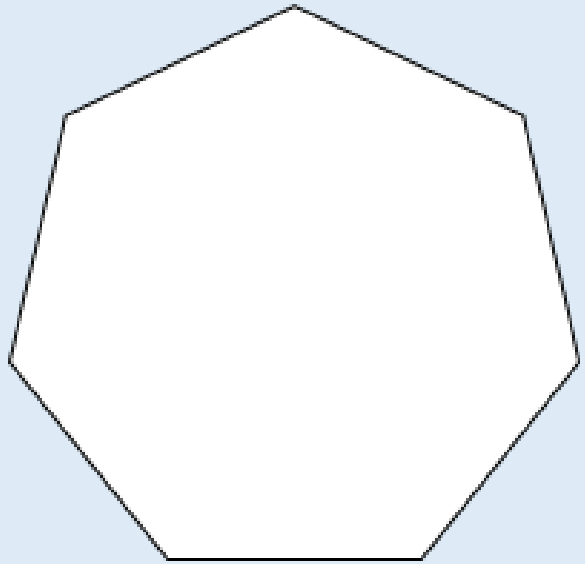
Mathematics of Origami

Origami makes it possible to construct geometrical constructions which are impossible to draw when using only a compass and a ruler.

trisectors



heptagon



Keywords

- **"Step"** – making one new fold

The number of steps required to fold a regular polygon depends on the folding technique used.

The number of steps required that we calculated, are not necessarily the actual **minimum** possible.

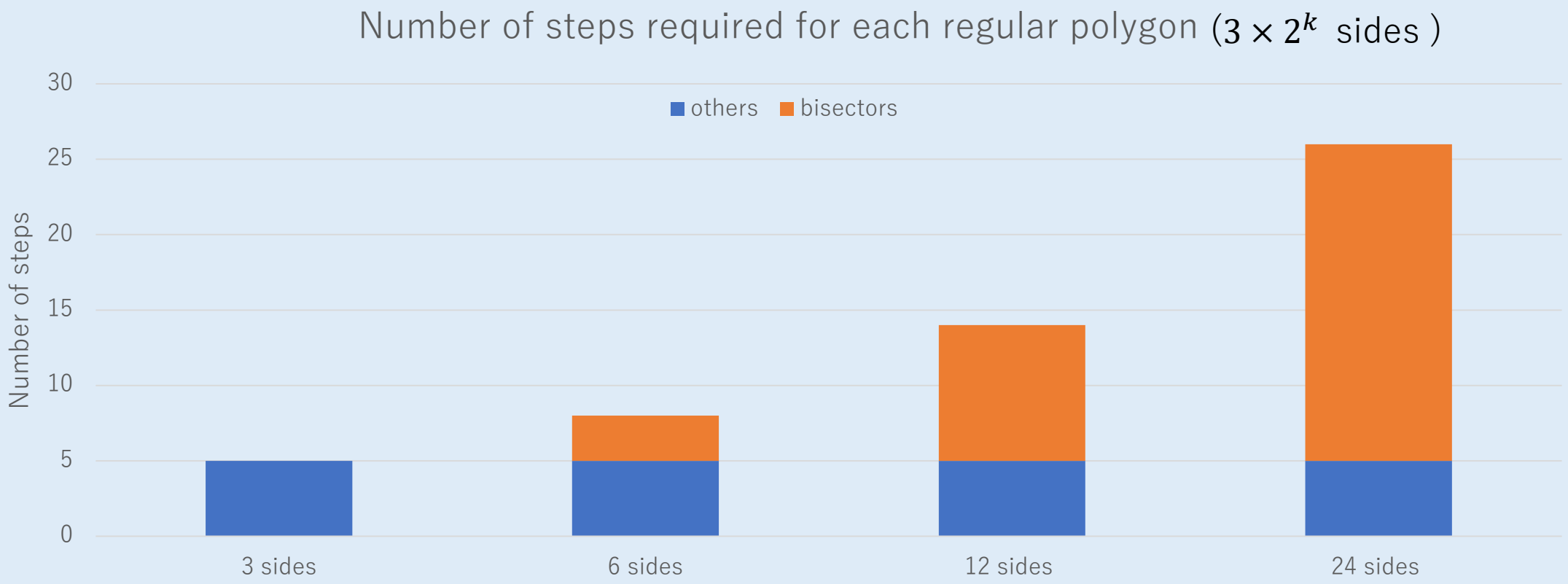
Goals

- Figuring out how many steps it would take to construct a regular polygon by focusing on how many sides it has

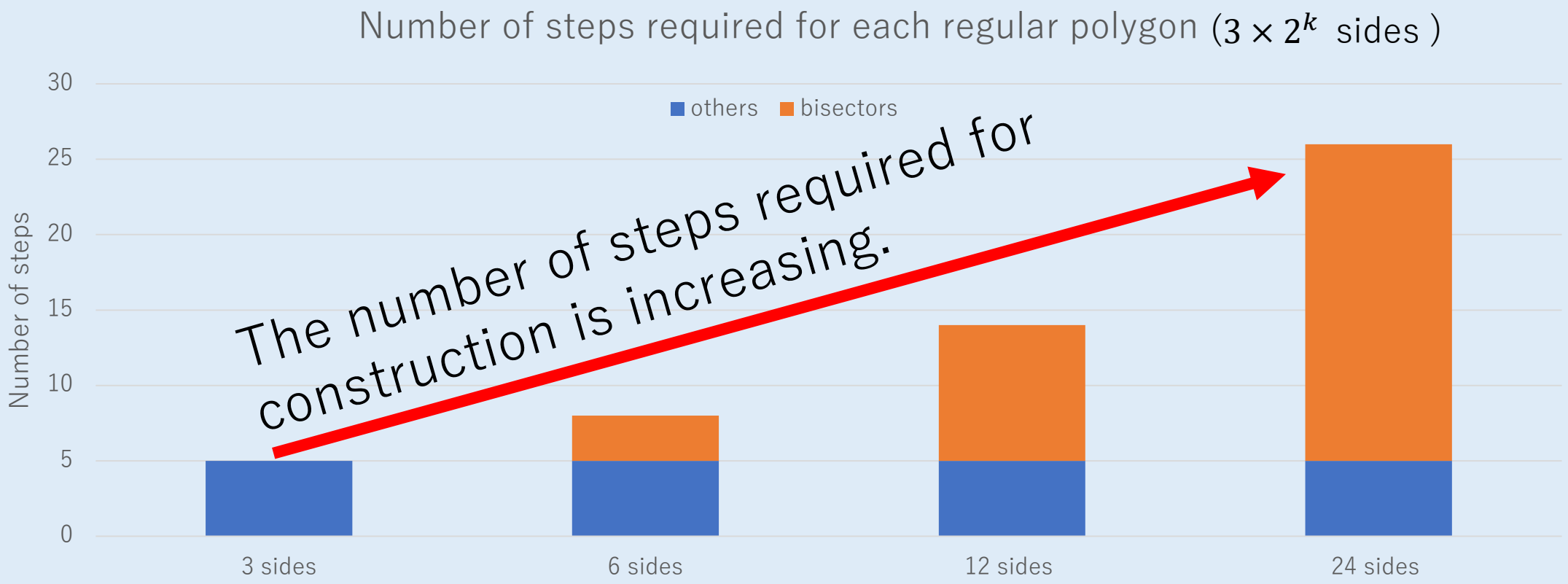
Research Topic 1 - Method

- Folded regular polygons by trial and error

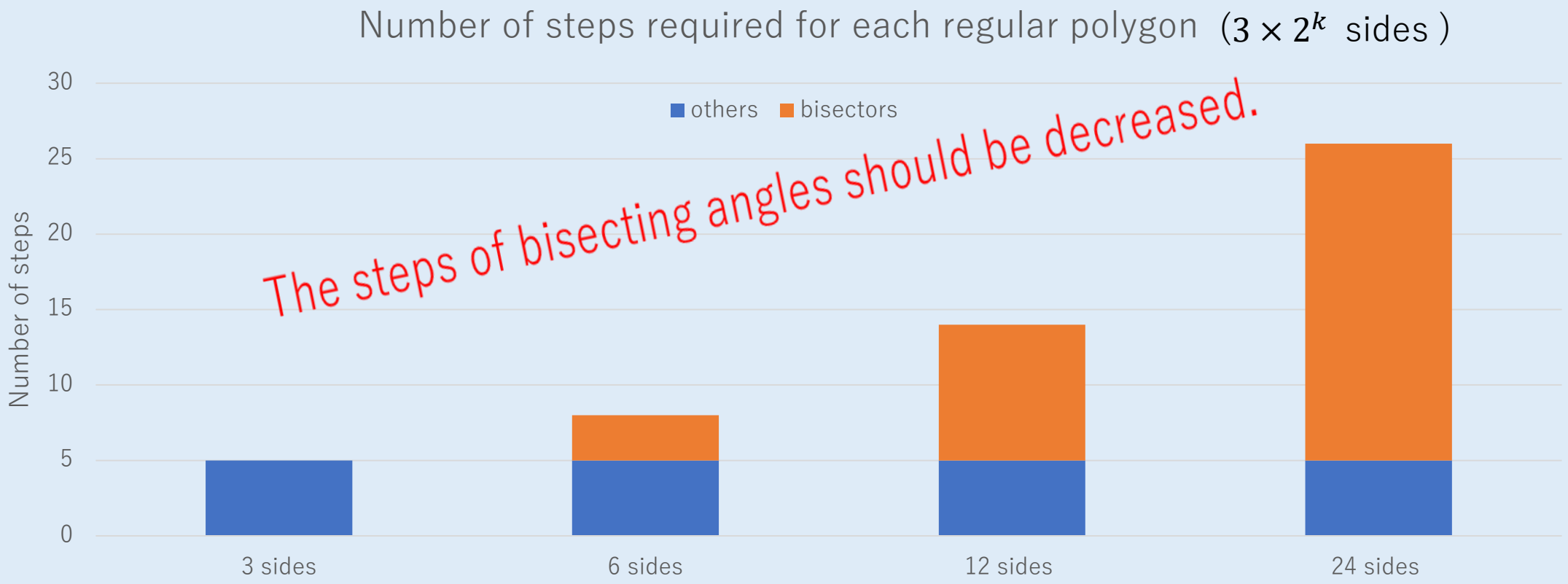
Results


Number of steps required for each regular polygon (3×2^k sides)
Number of steps
30
25
20
15
10
5
0
others
bisectors
3 sides
6 sides
12 sides
24 sides

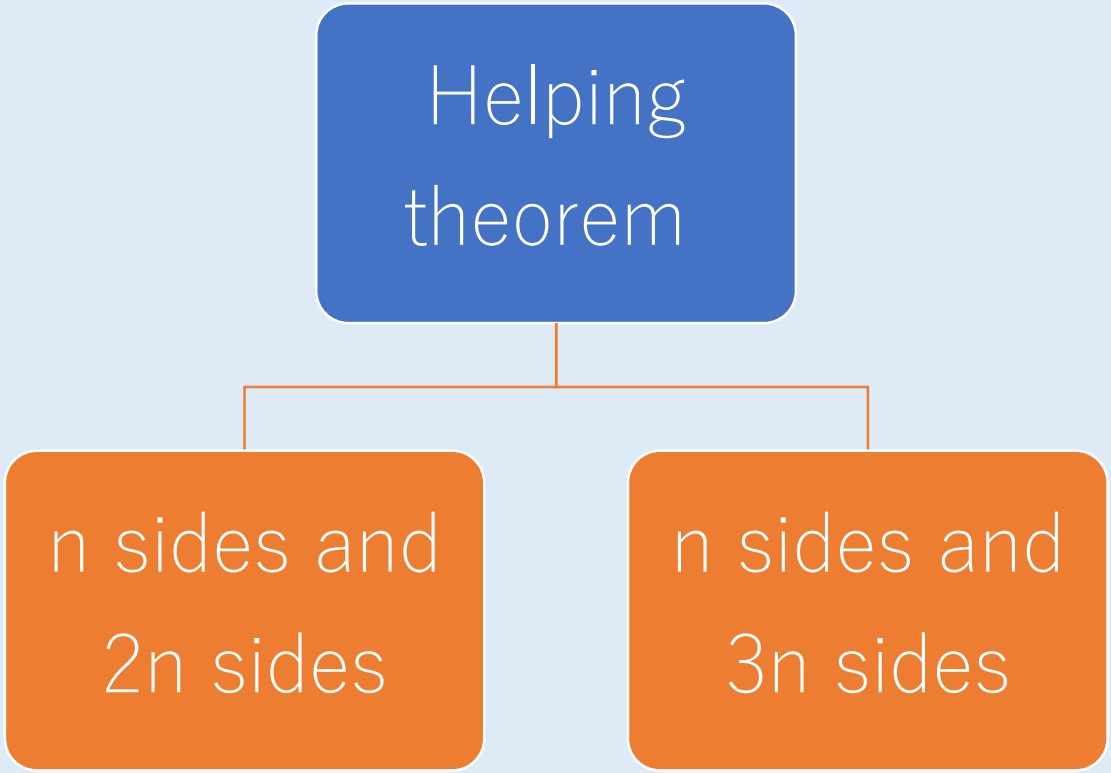
Results


Number of steps required for each regular polygon (3×2^k sides)
Number of steps
30
25
20
15
10
5
0
others
bisectors
The number of steps required for construction is increasing.
3 sides
6 sides
12 sides
24 sides

Results


Number of steps required for each regular polygon (3 × 2^k sides)
Number of steps
30
25
20
15
10
5
0
others
bisectors
The steps of bisecting angles should be decreased.
3 sides
6 sides
12 sides
24 sides

Next topic

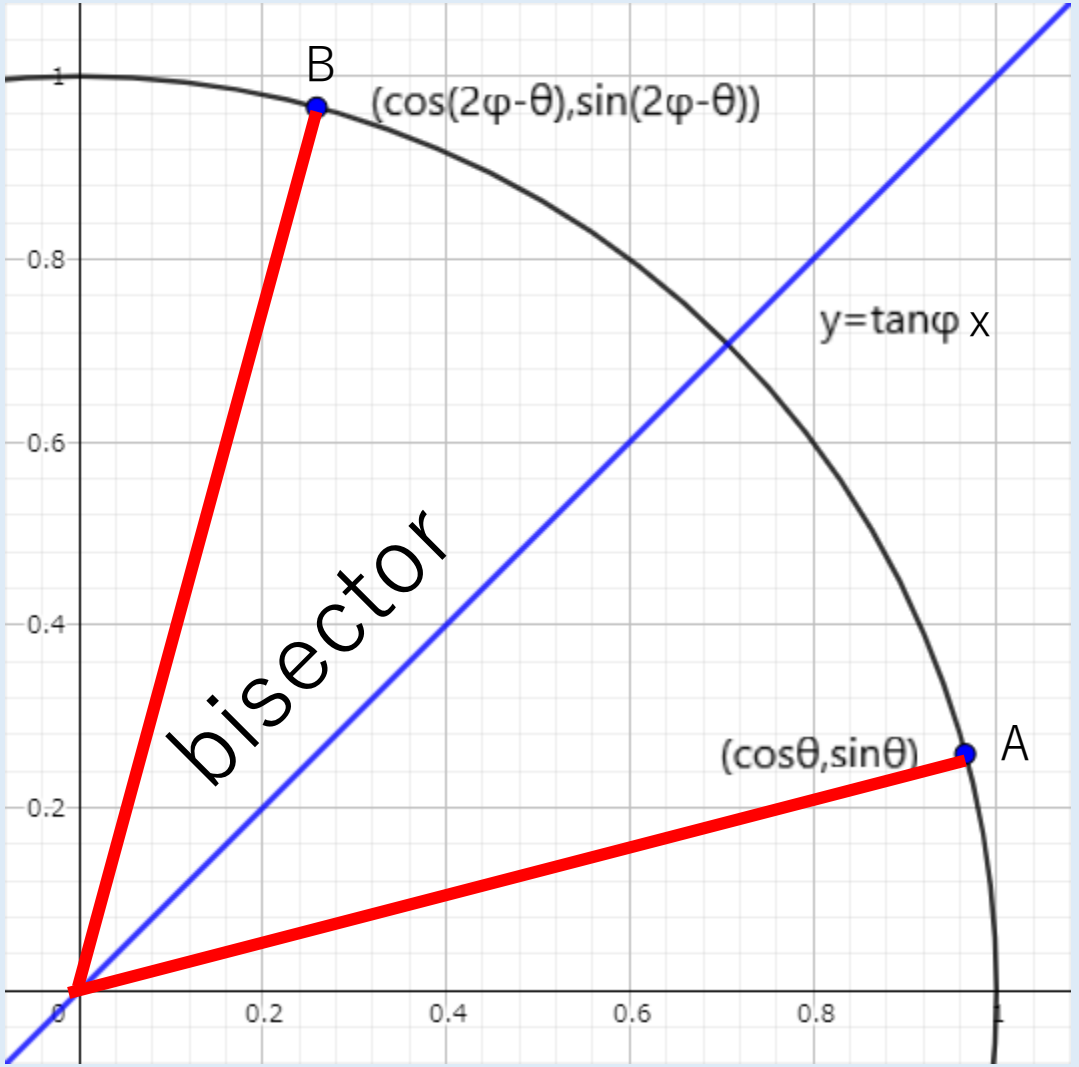


Discussion (helping theorem)

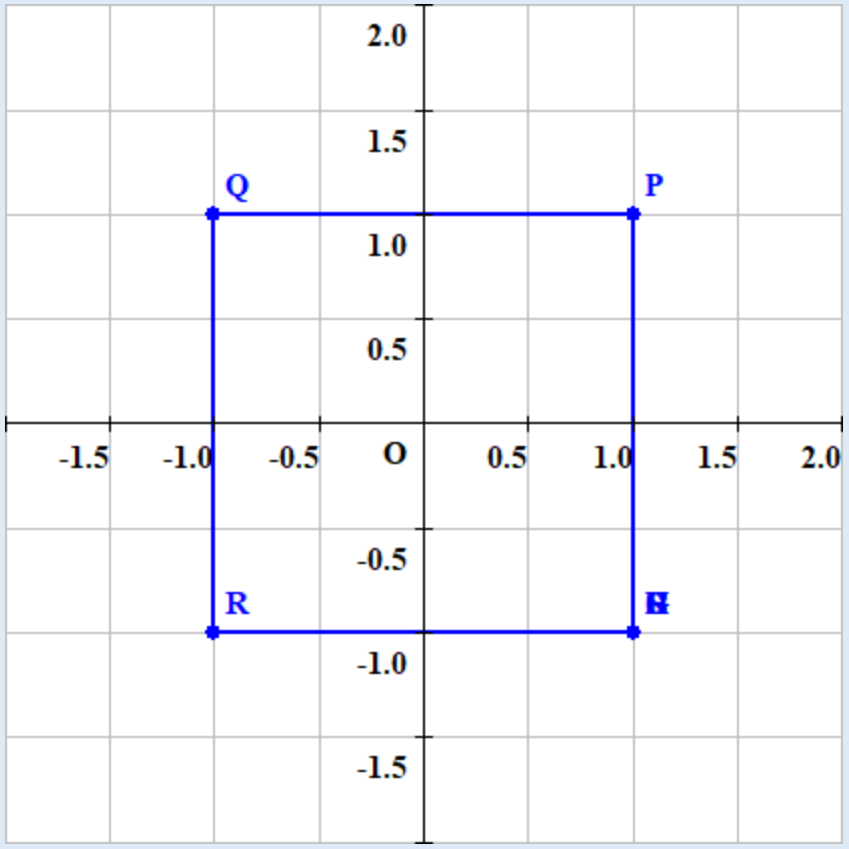
$(\cos\theta, \sin\theta)$ on circle $x^2 + y^2 = 1$

$y = (\tan\varphi)x$
Folded

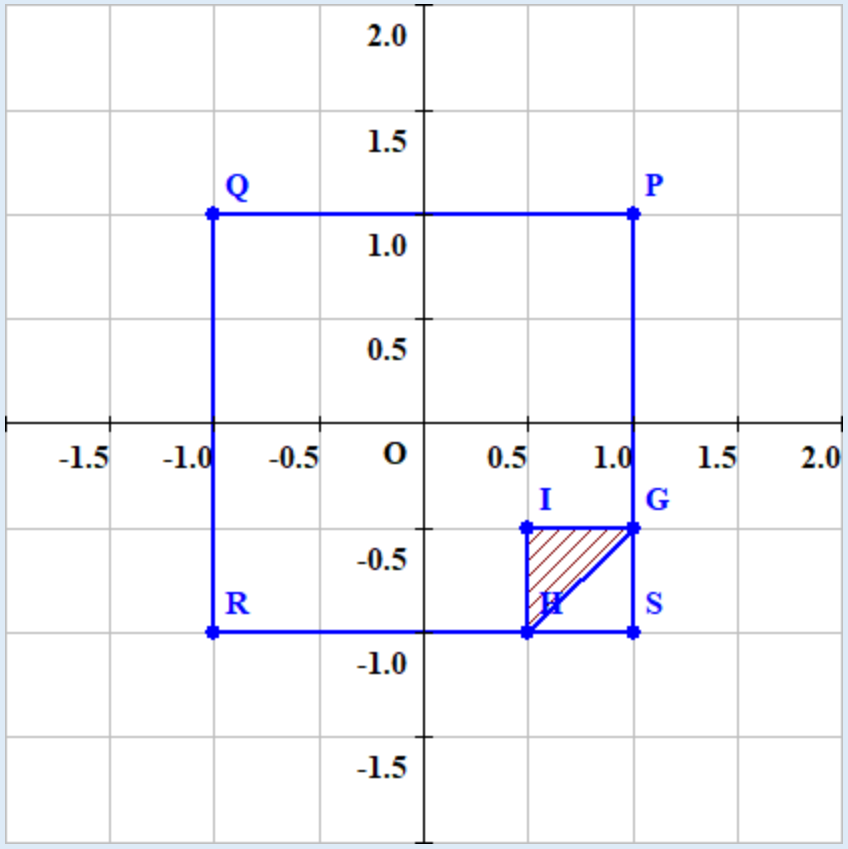
$(\cos(2\varphi - \theta), \sin(2\varphi - \theta))$



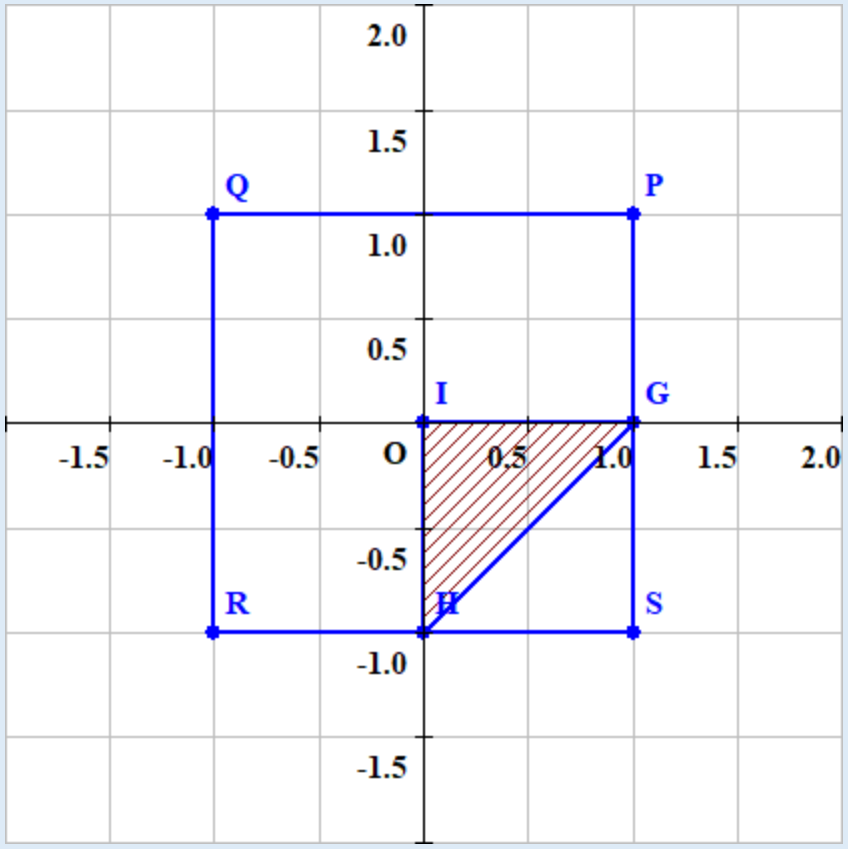
Bisecting motion



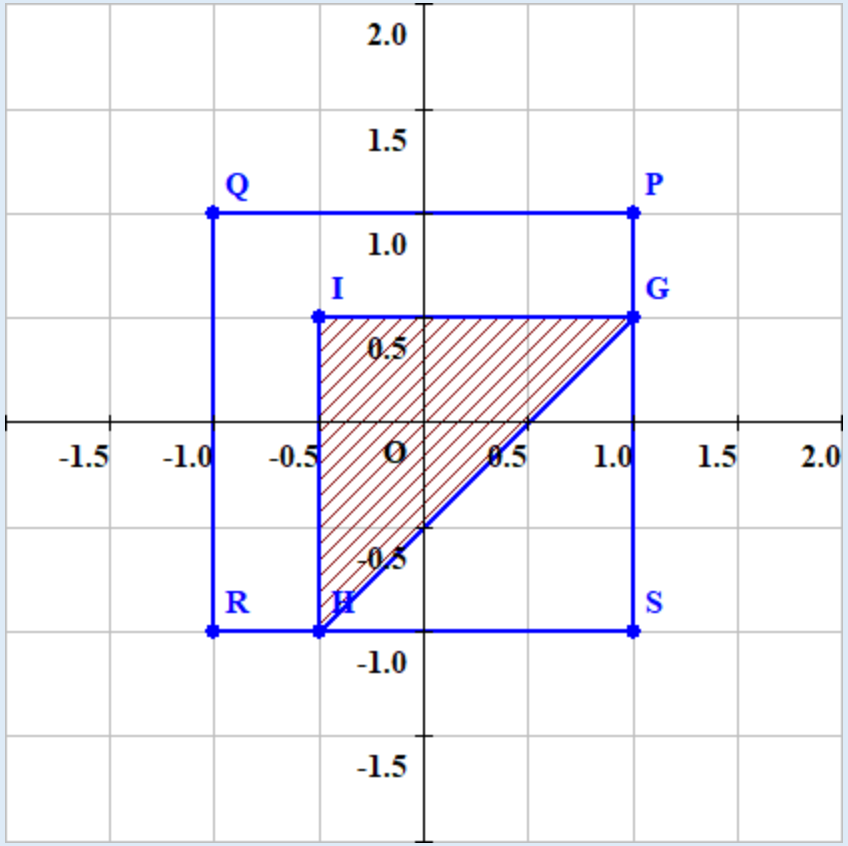
Bisecting motion



Bisecting motion



Bisecting motion



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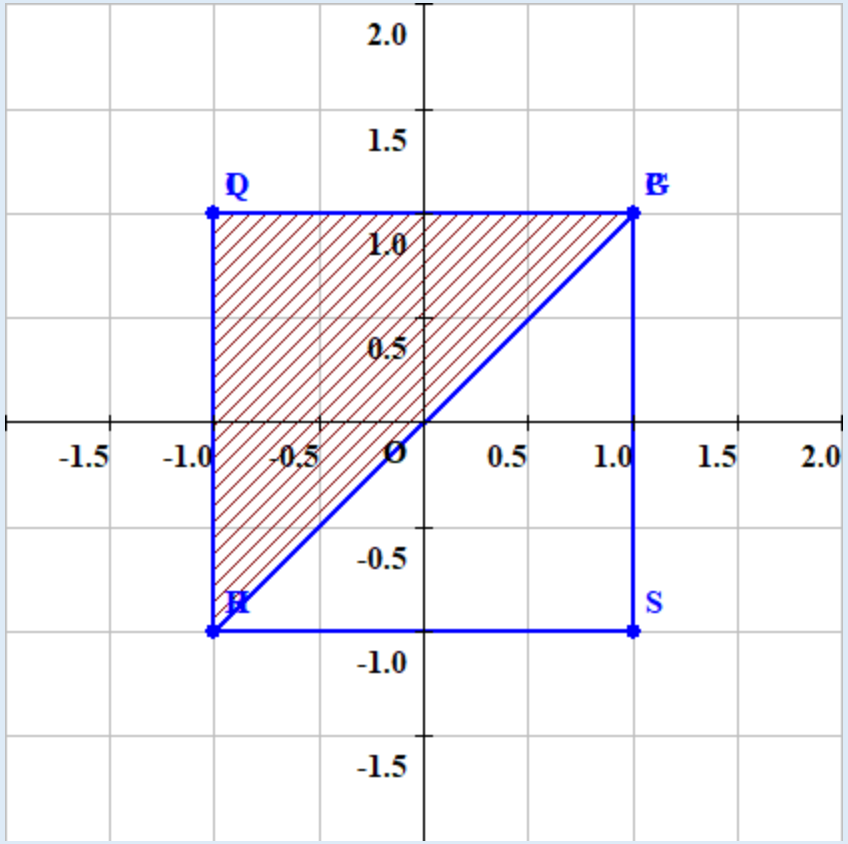
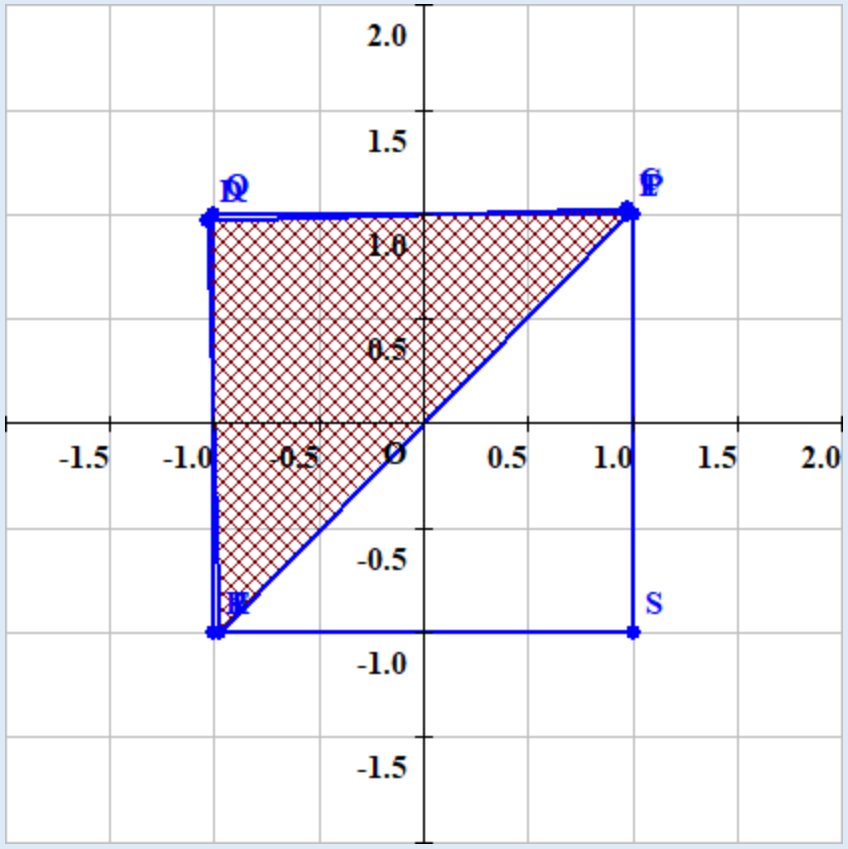


Bisecting motion



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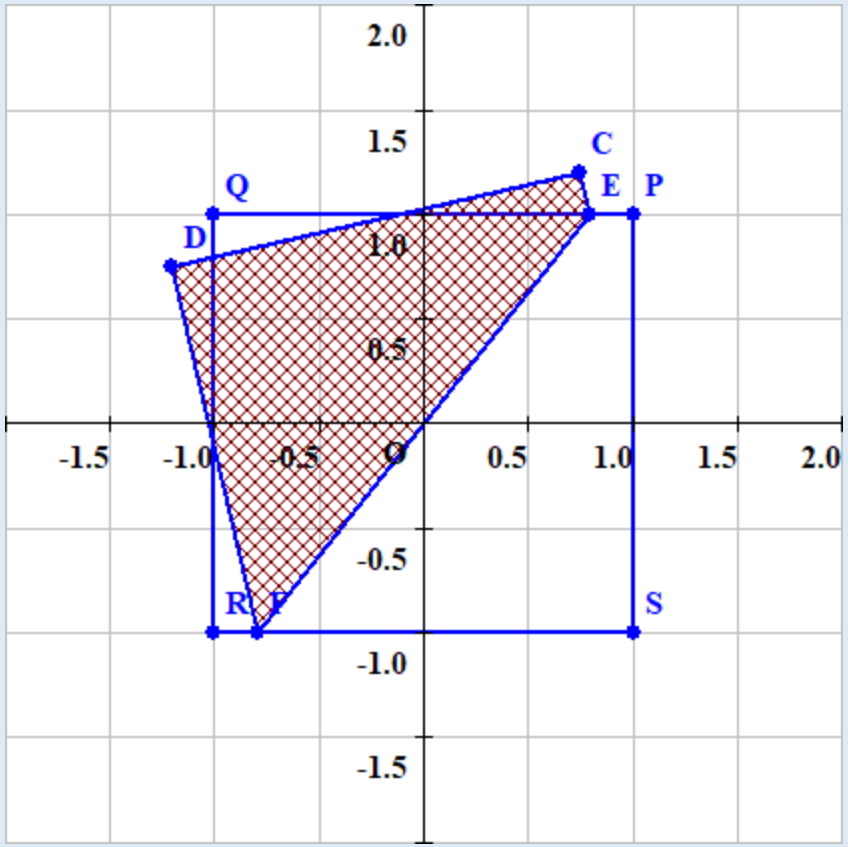

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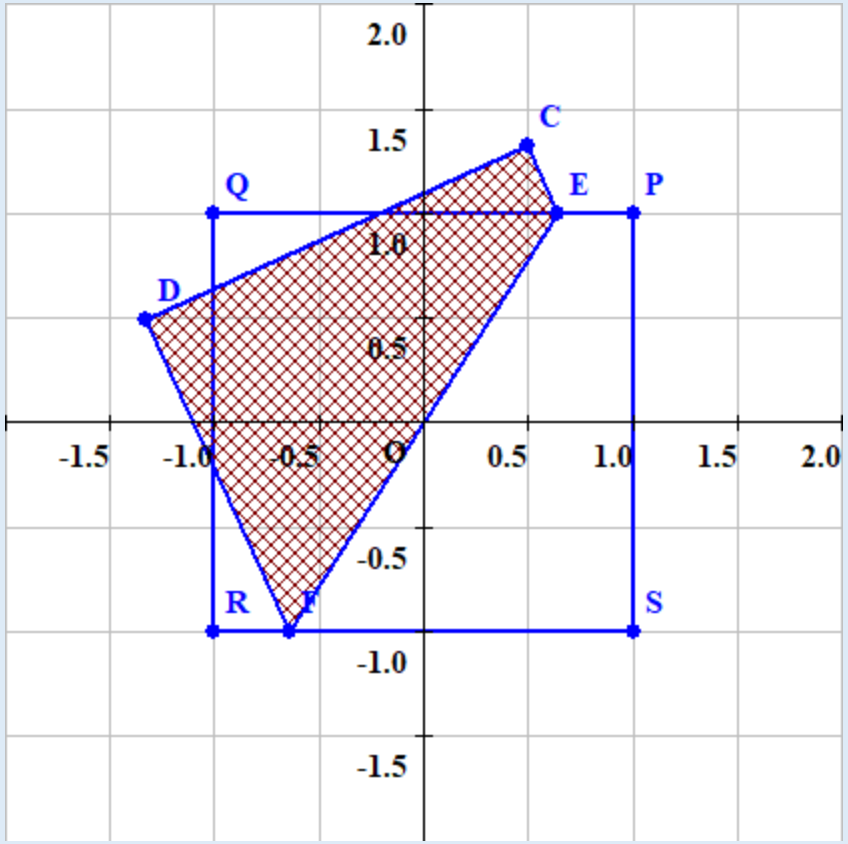
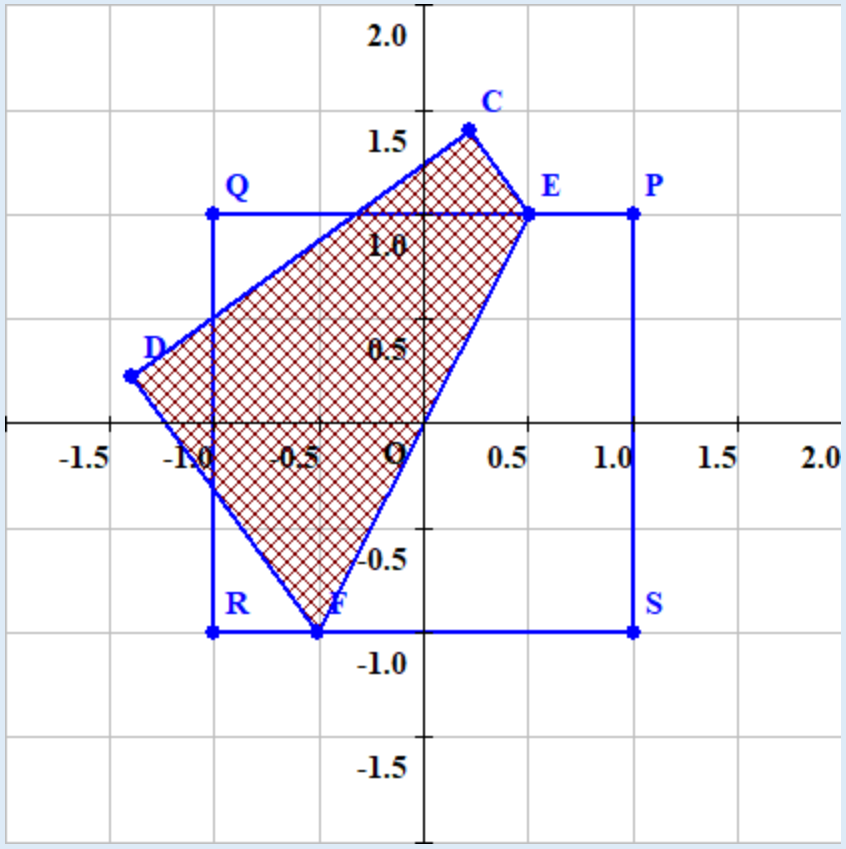
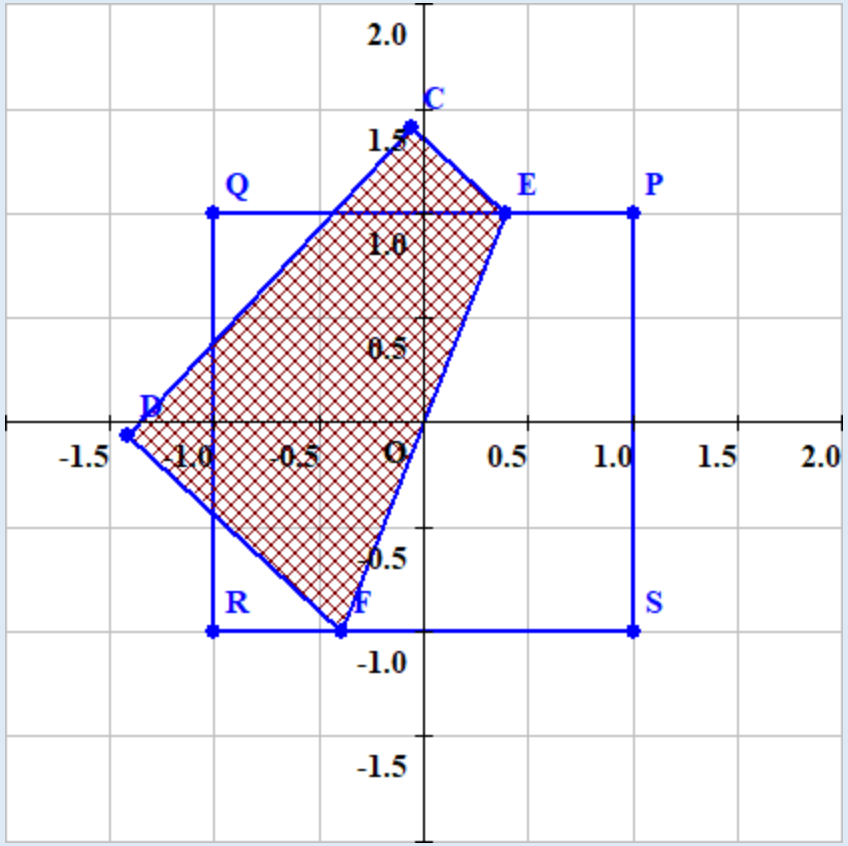


Bisecting motion



Bisecting motion



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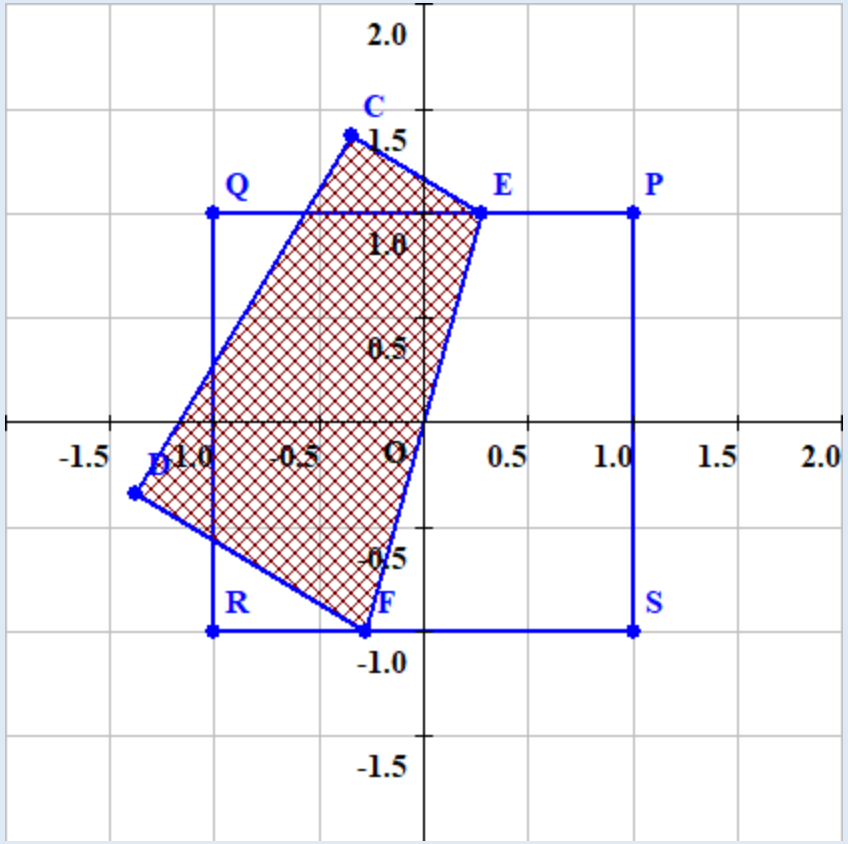

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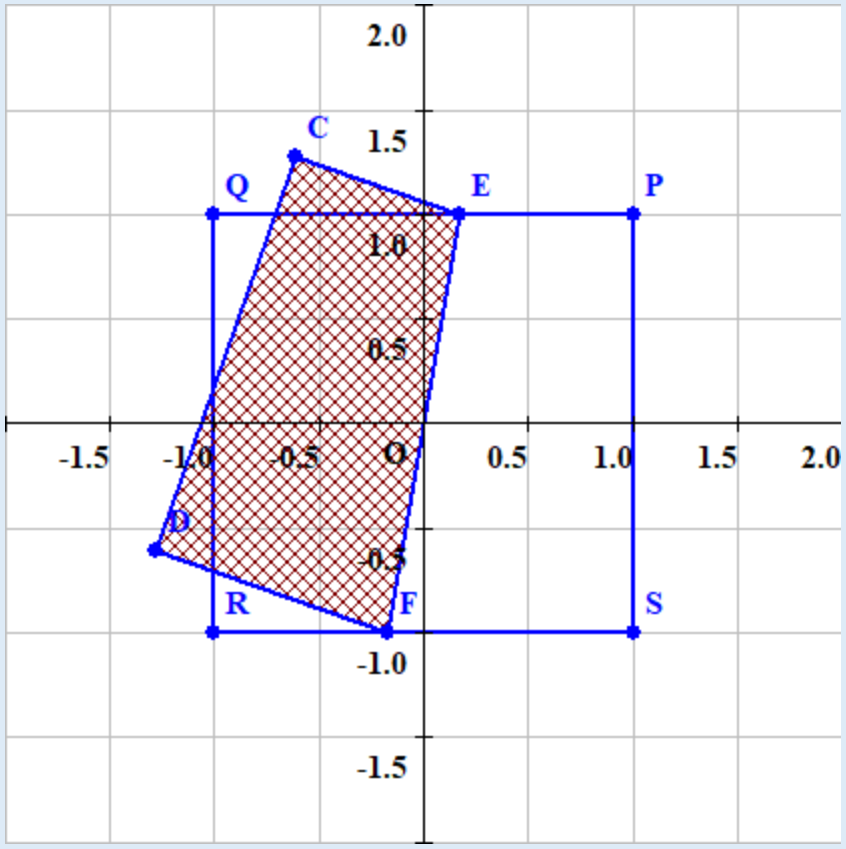

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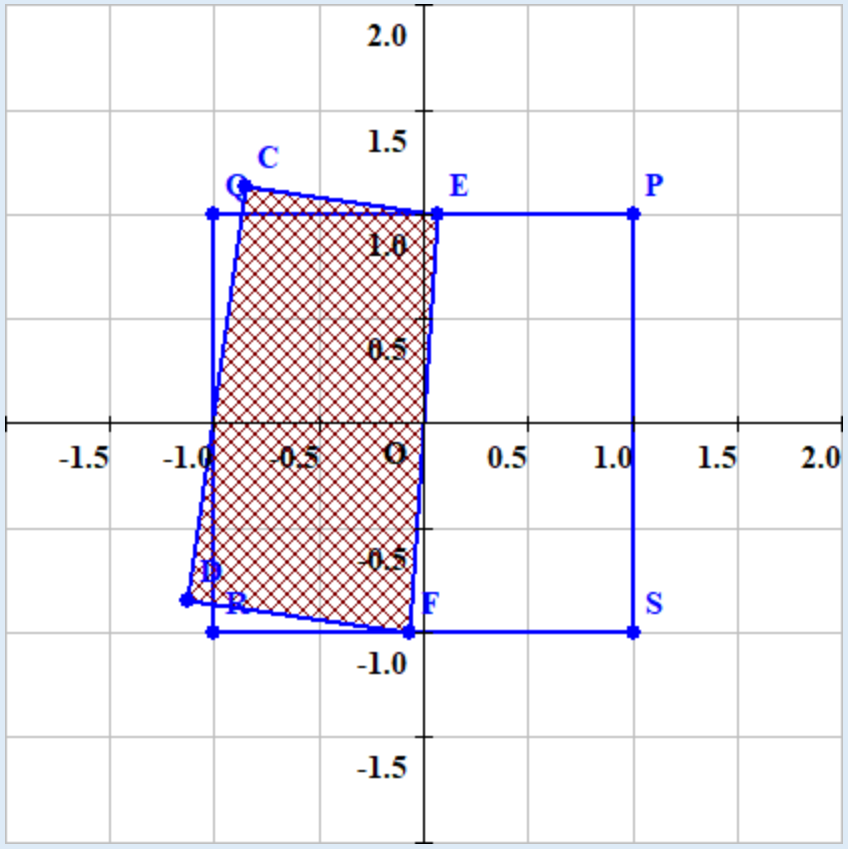

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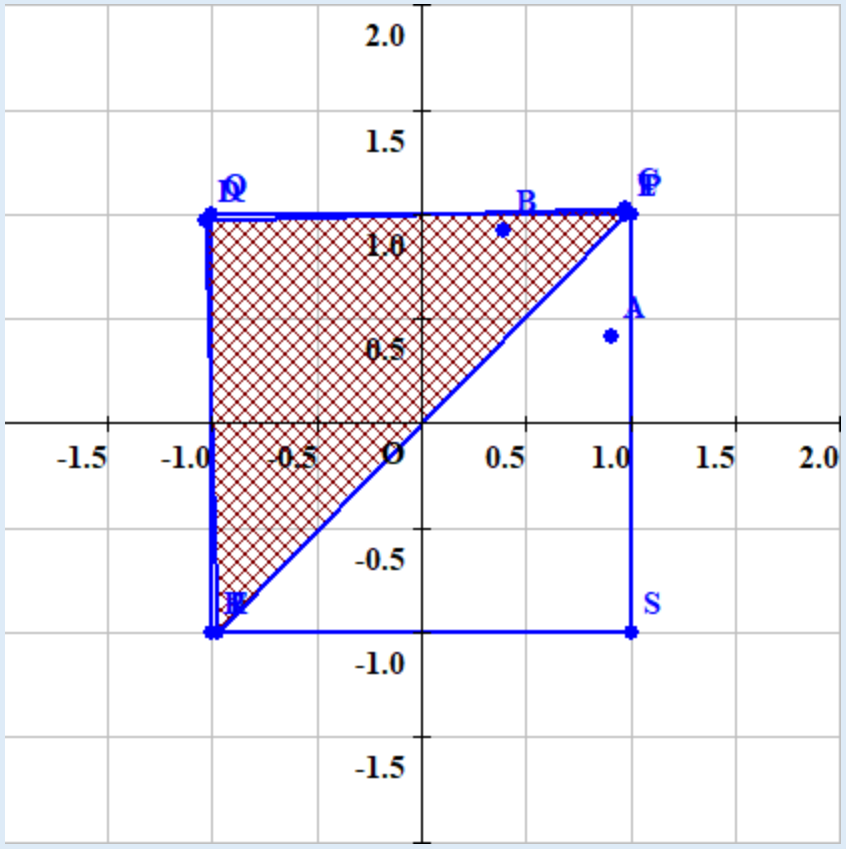

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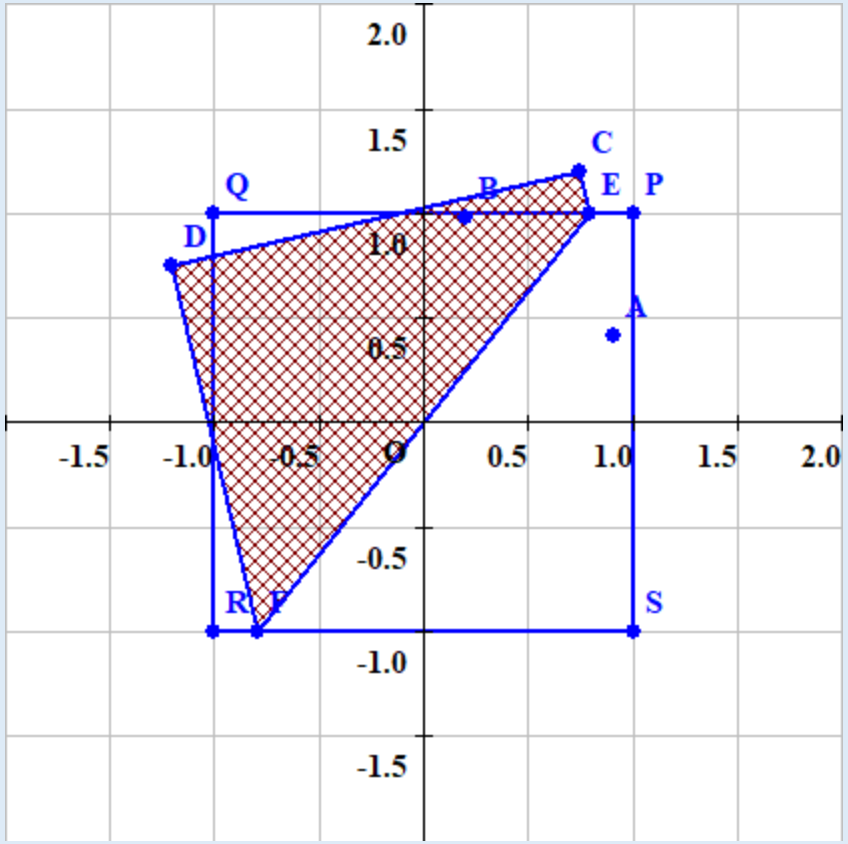

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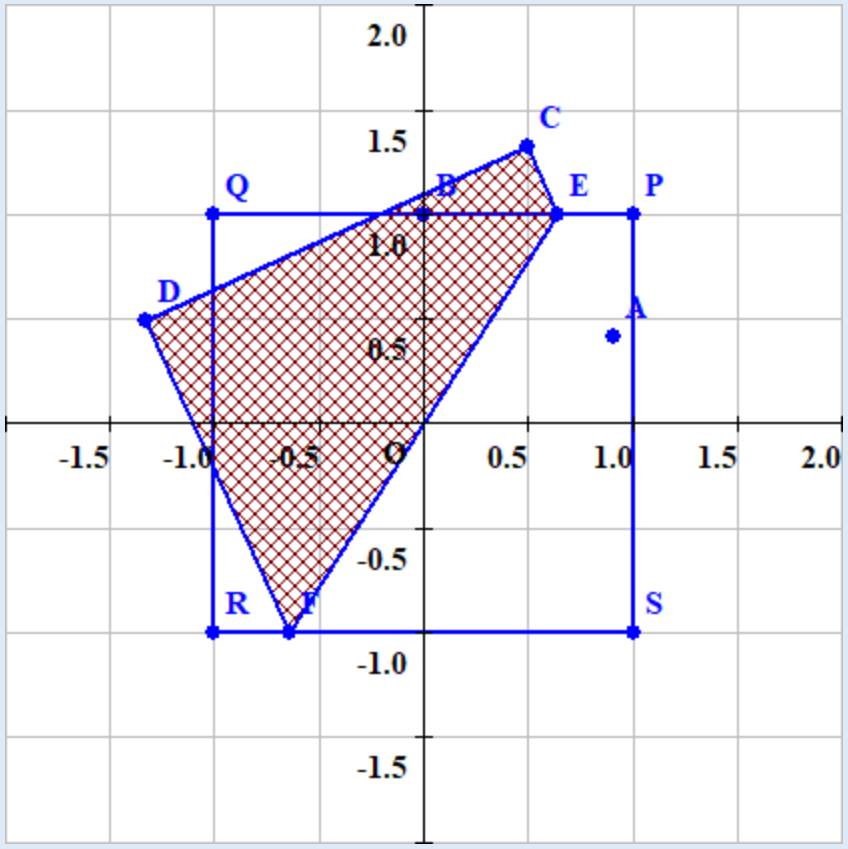

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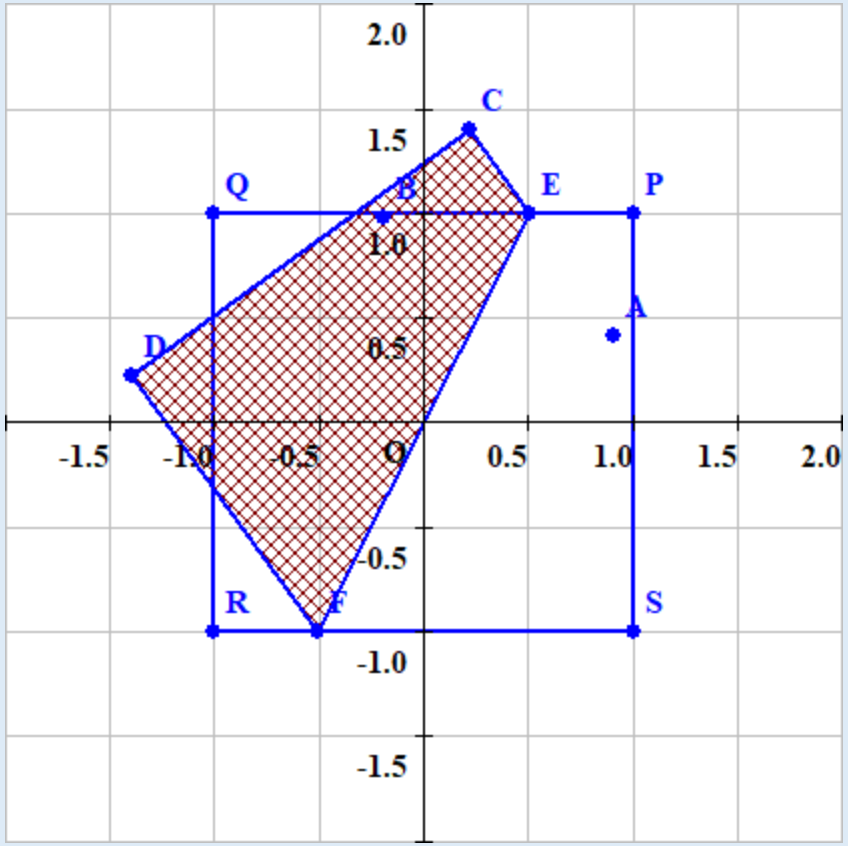

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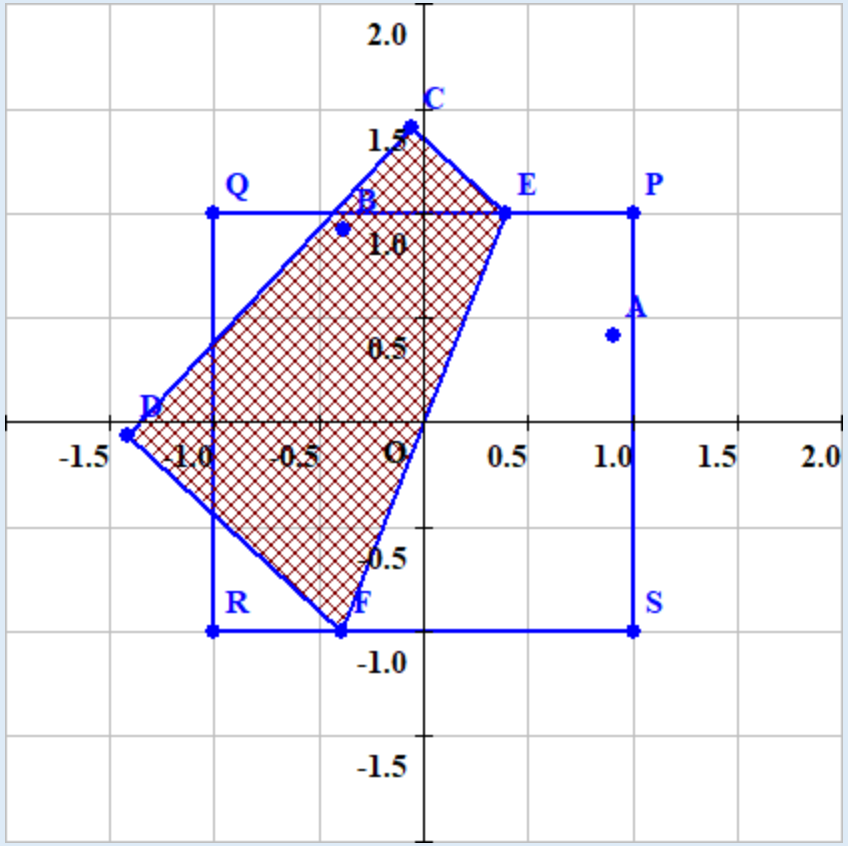

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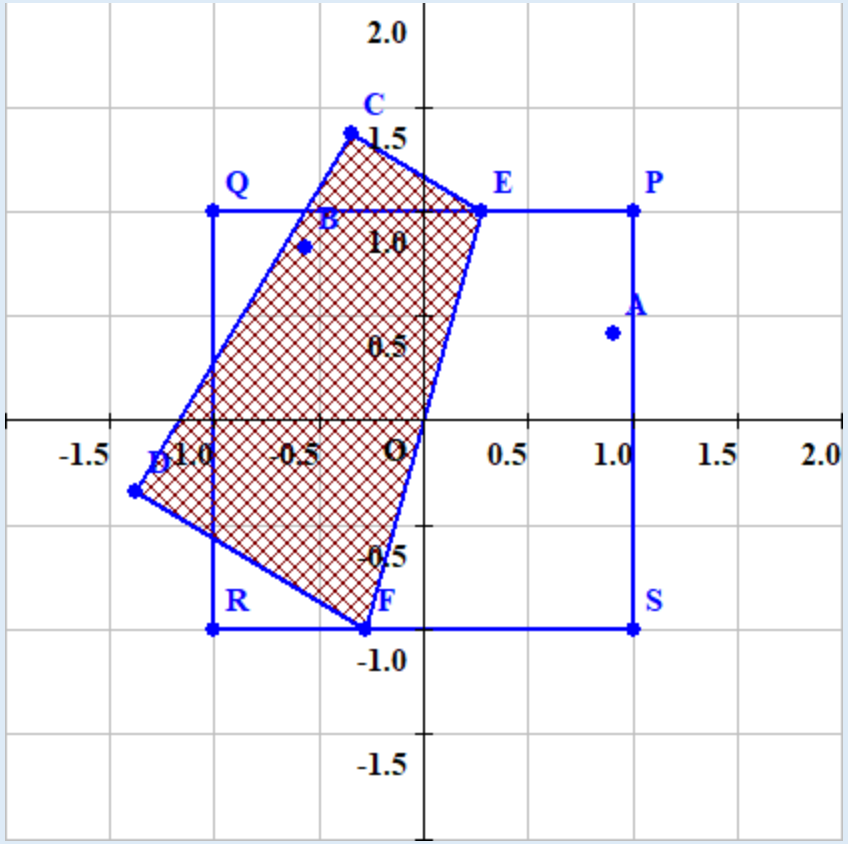

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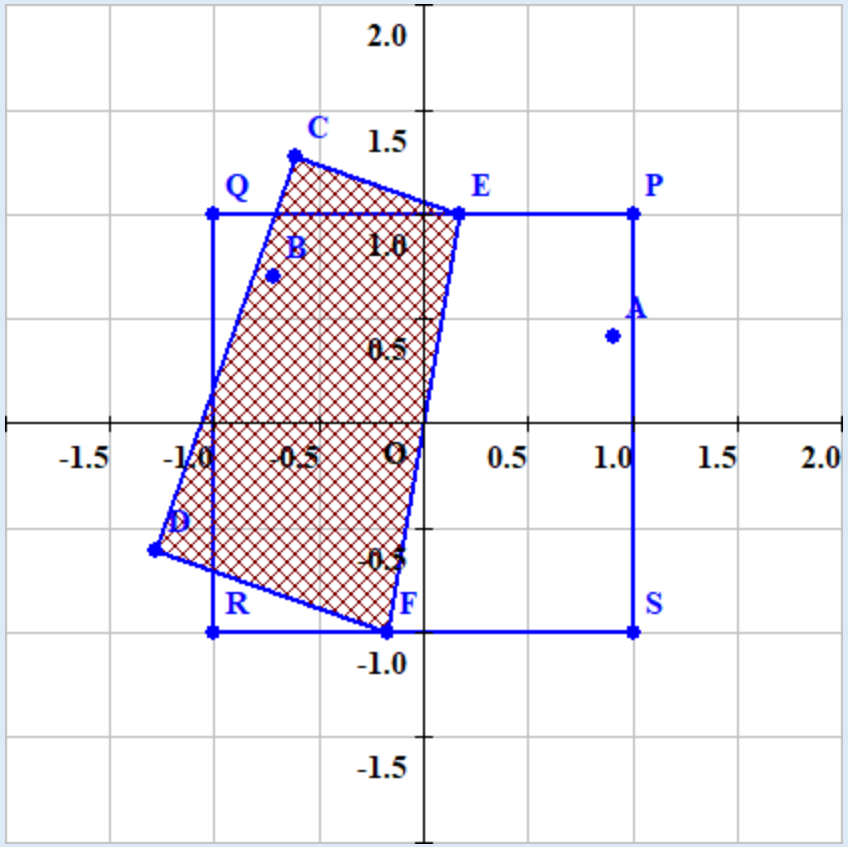

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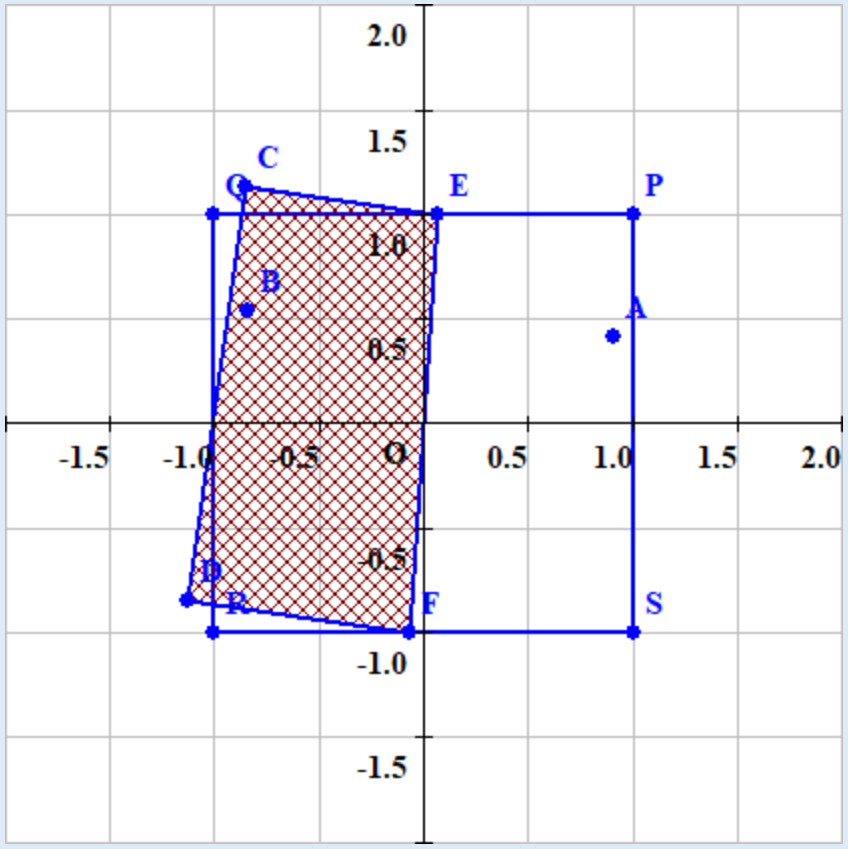


Discussion (generalization)

The line

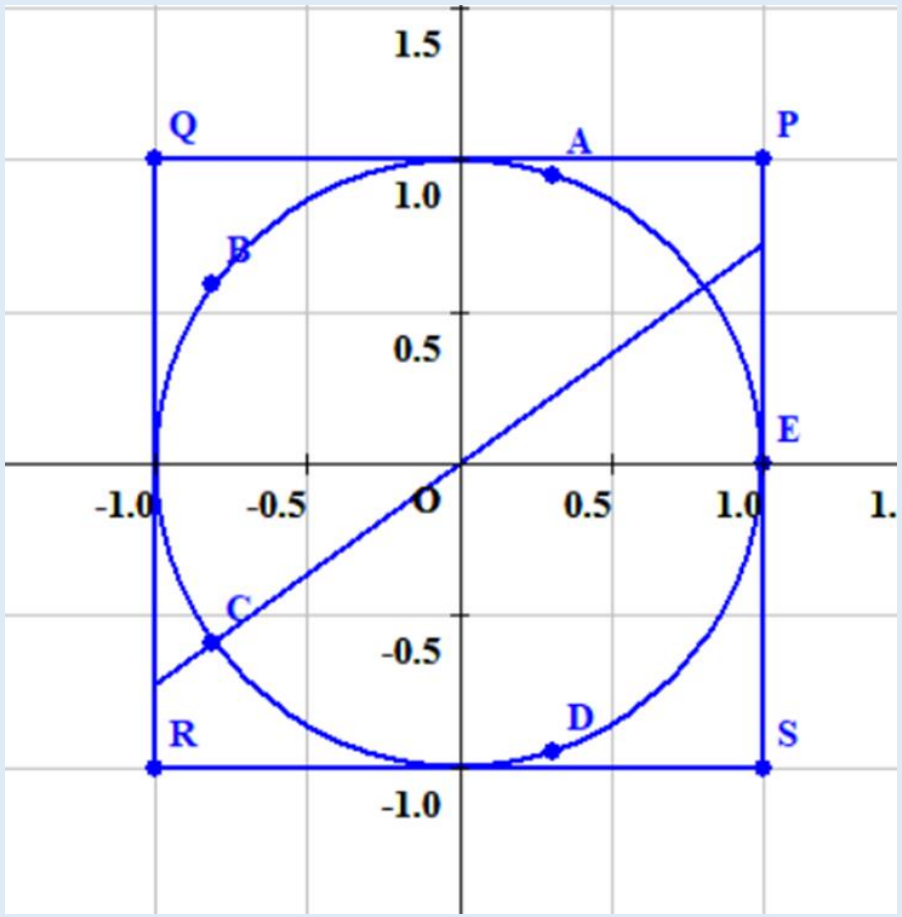
$$y = \left(\tan\frac{2\pi}{4p}\right)x$$

and the regular polygon with p sides

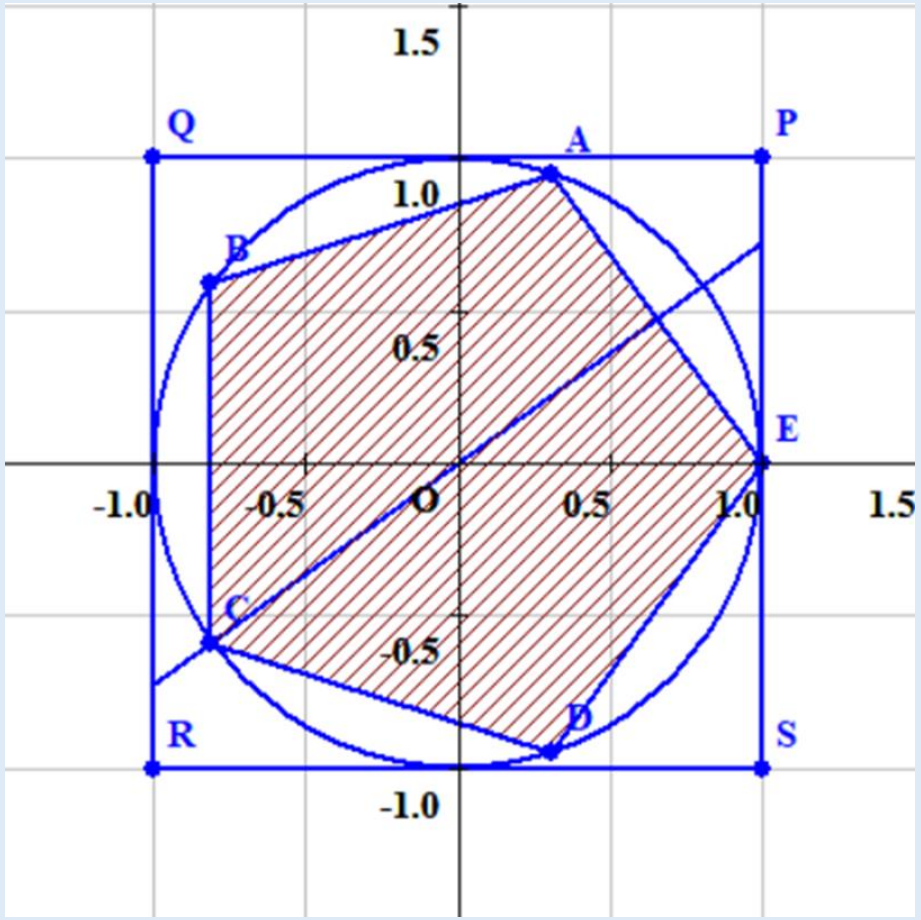
1 step

The regular polygons with 2p sides

Discussion (generalization)



Discussion (generalization)



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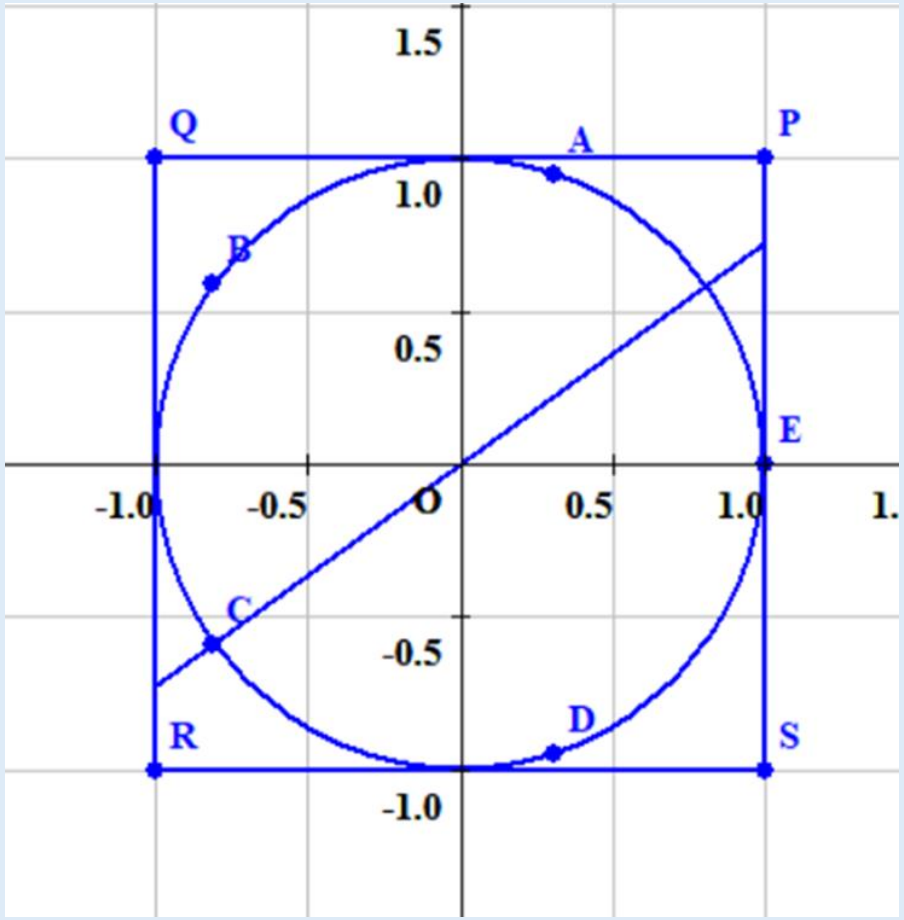
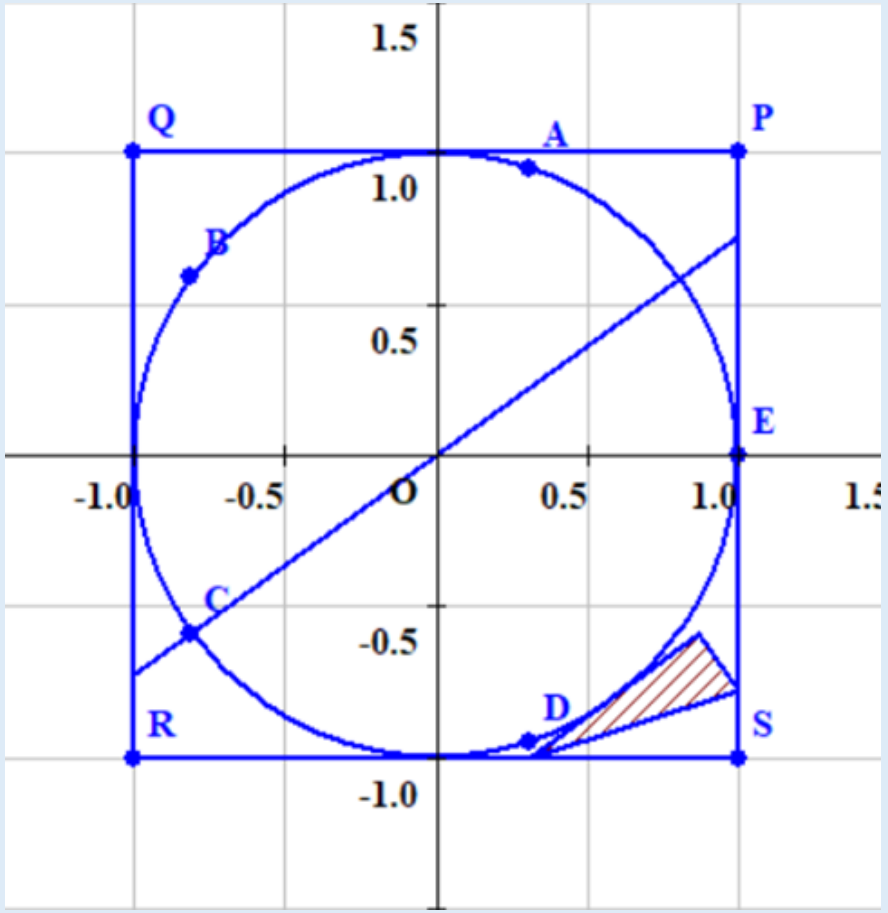
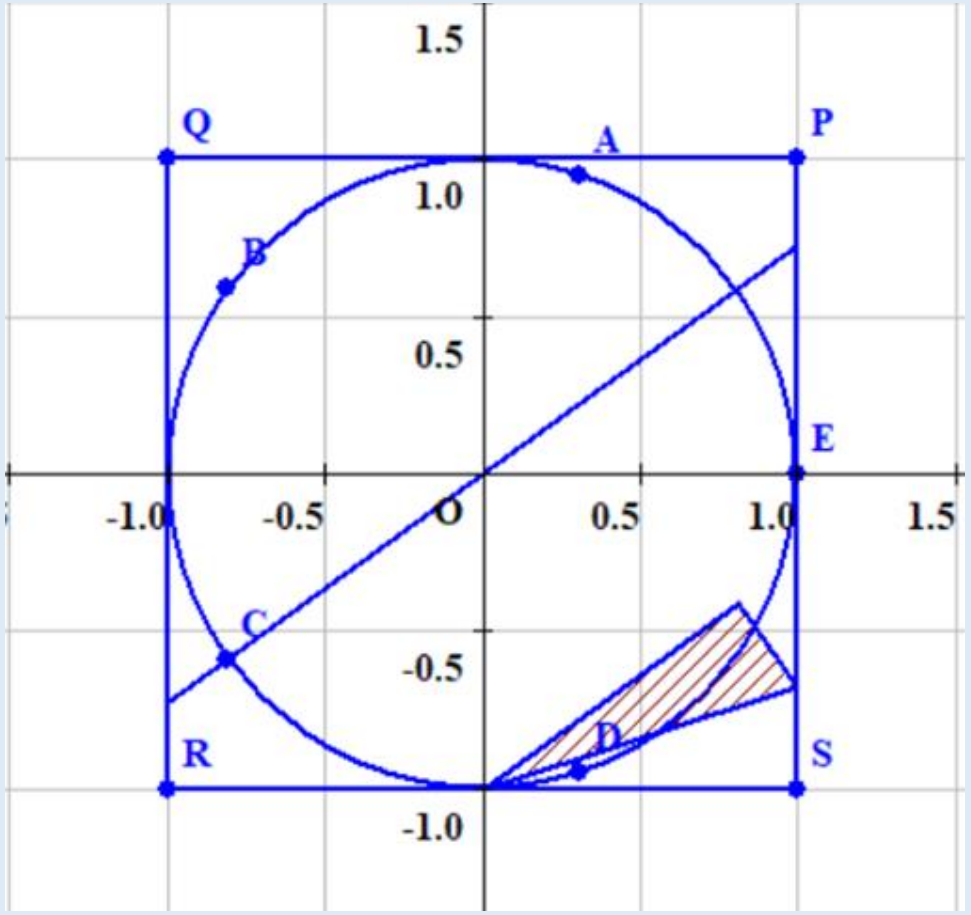


Discussion (generalization)



Discussion (generalization)



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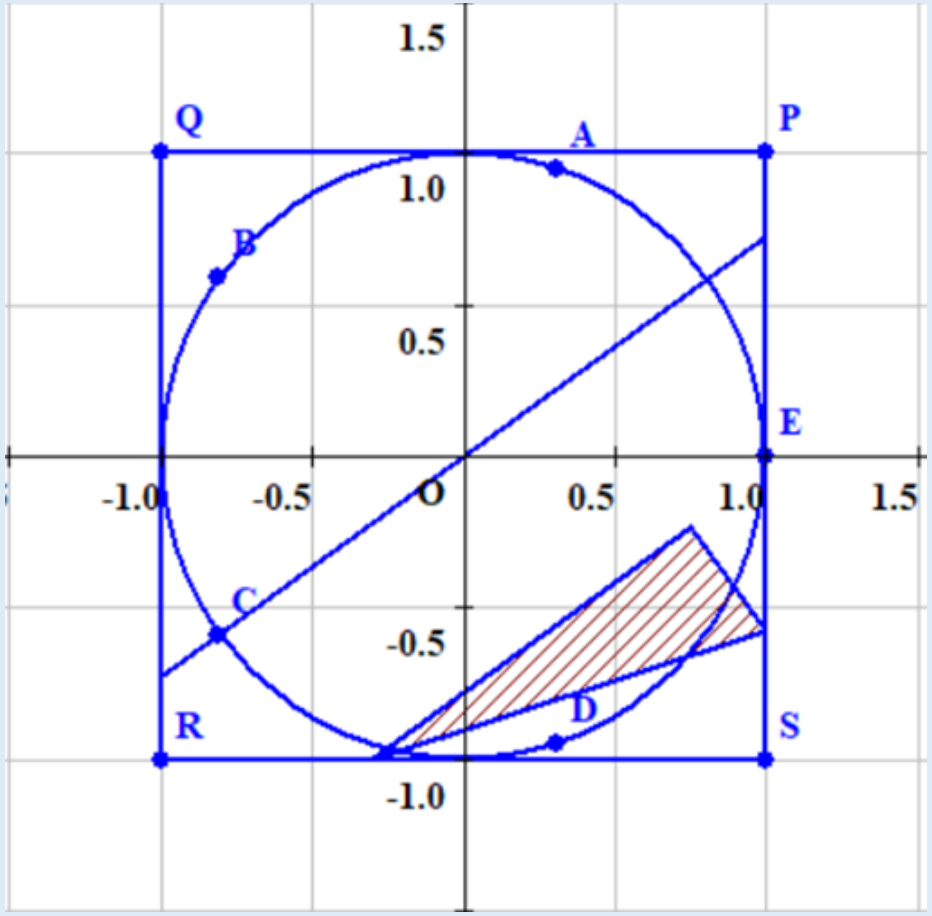
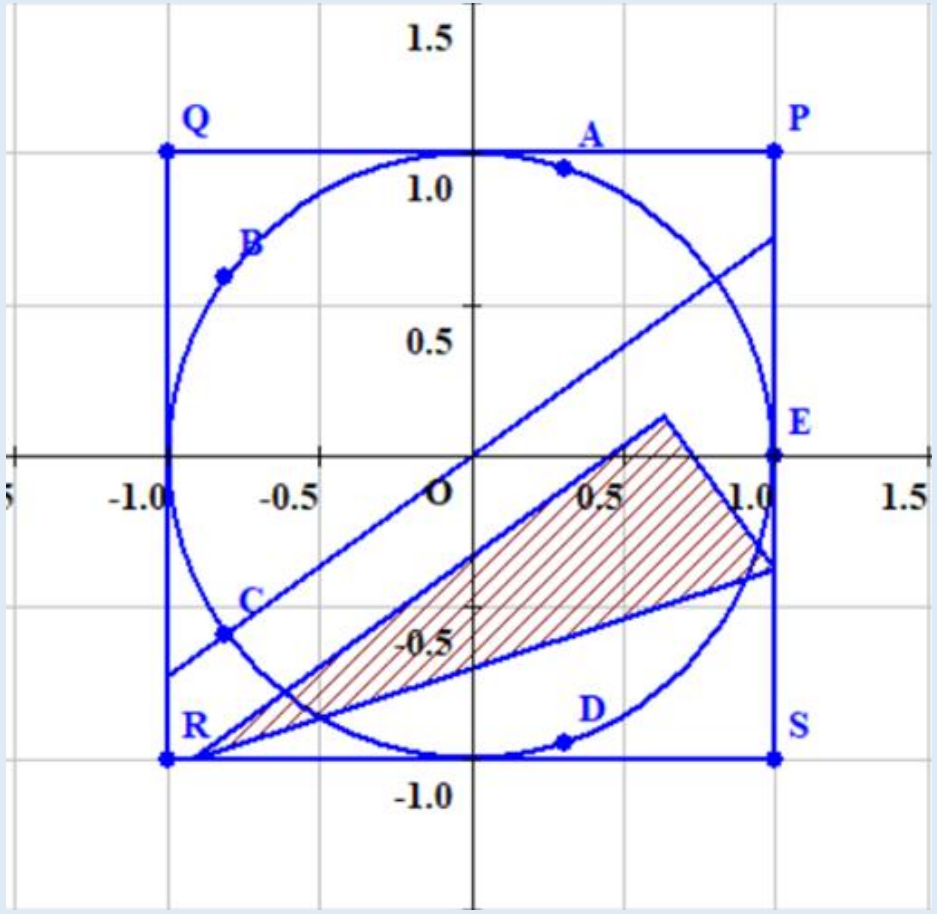


Discussion (generalization)



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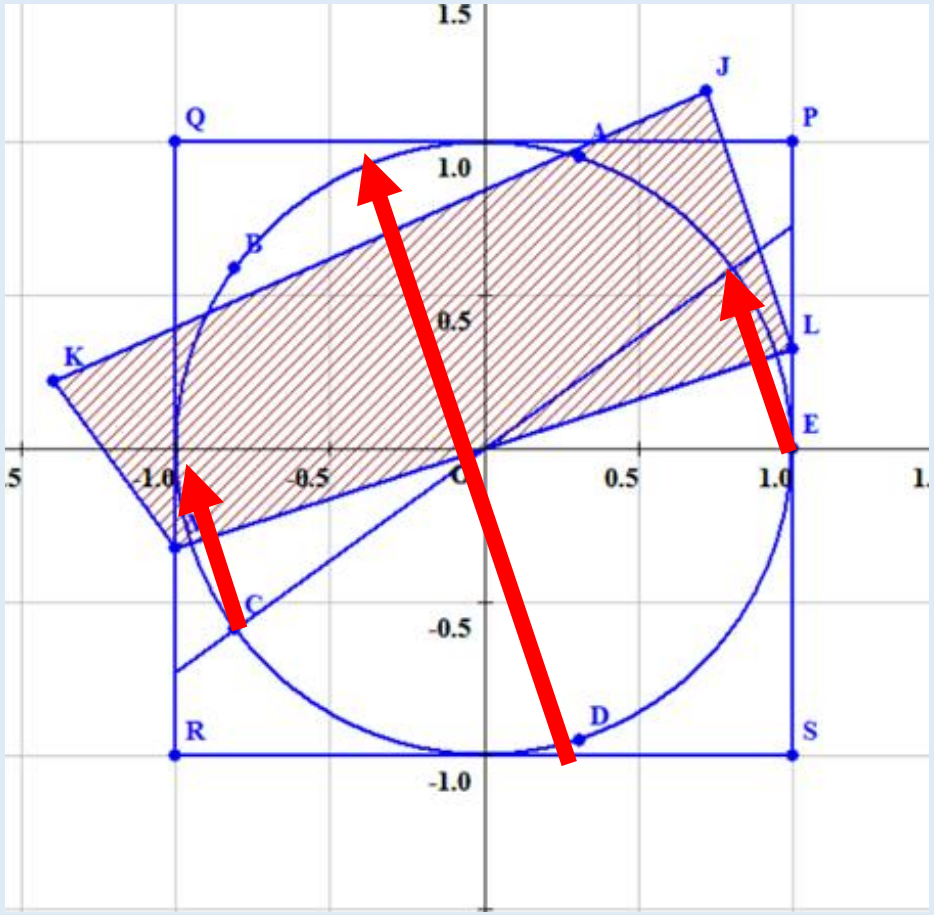
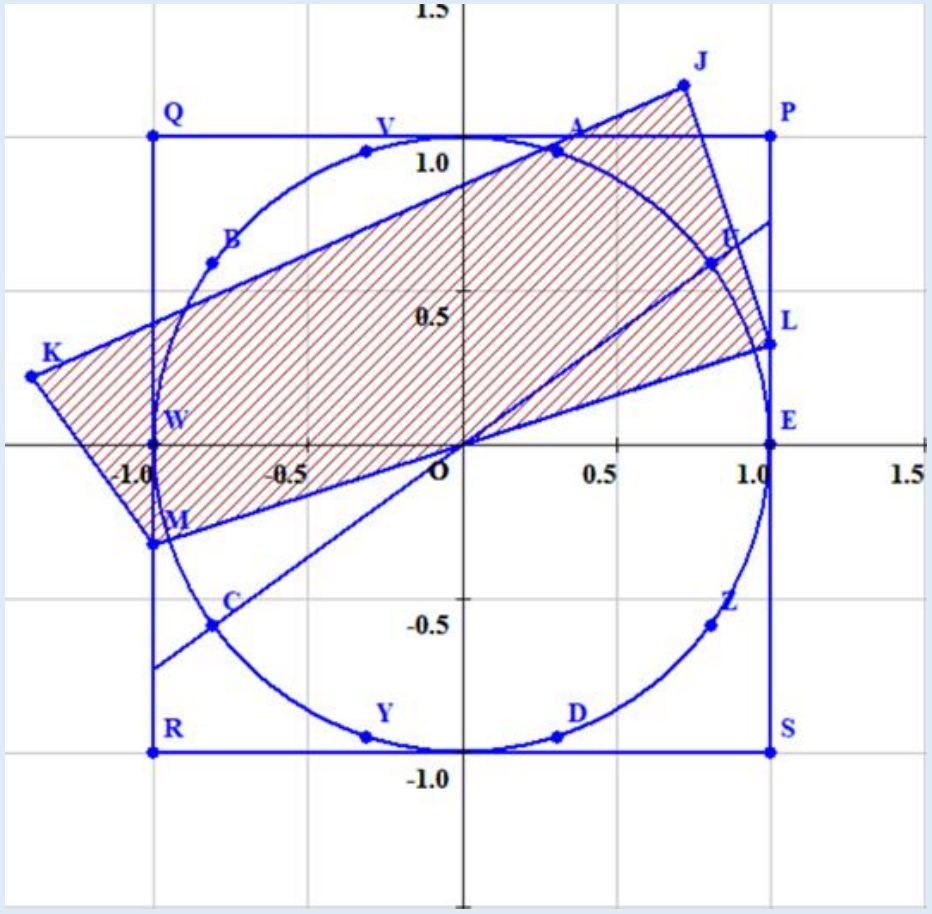
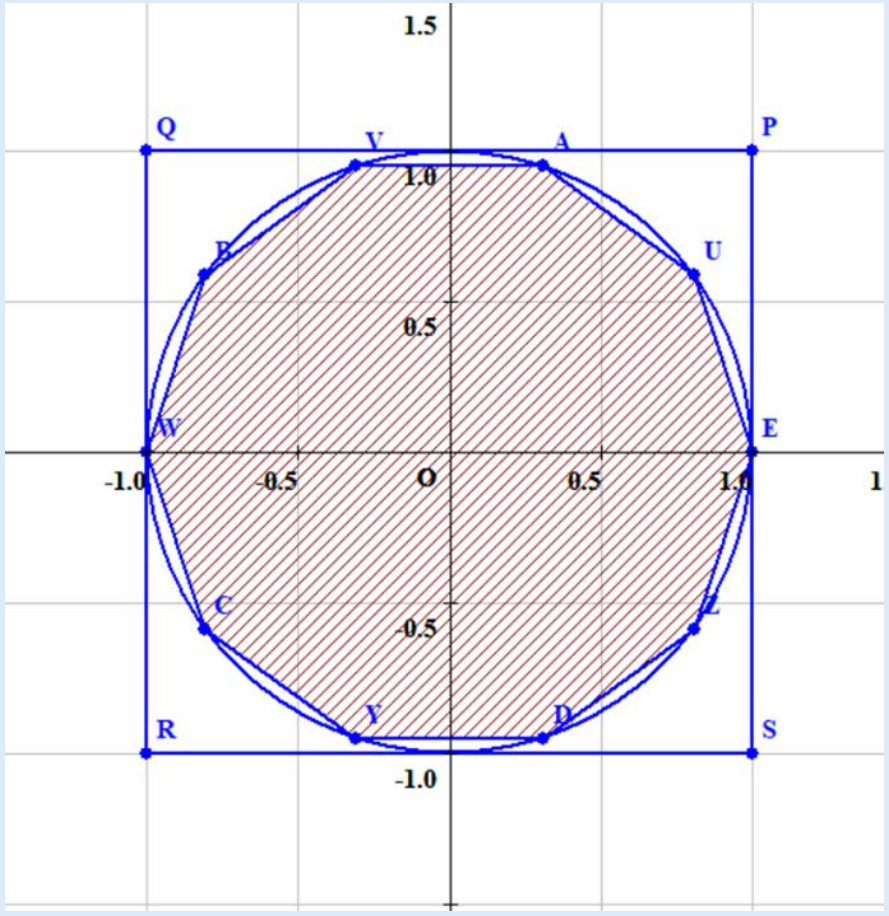


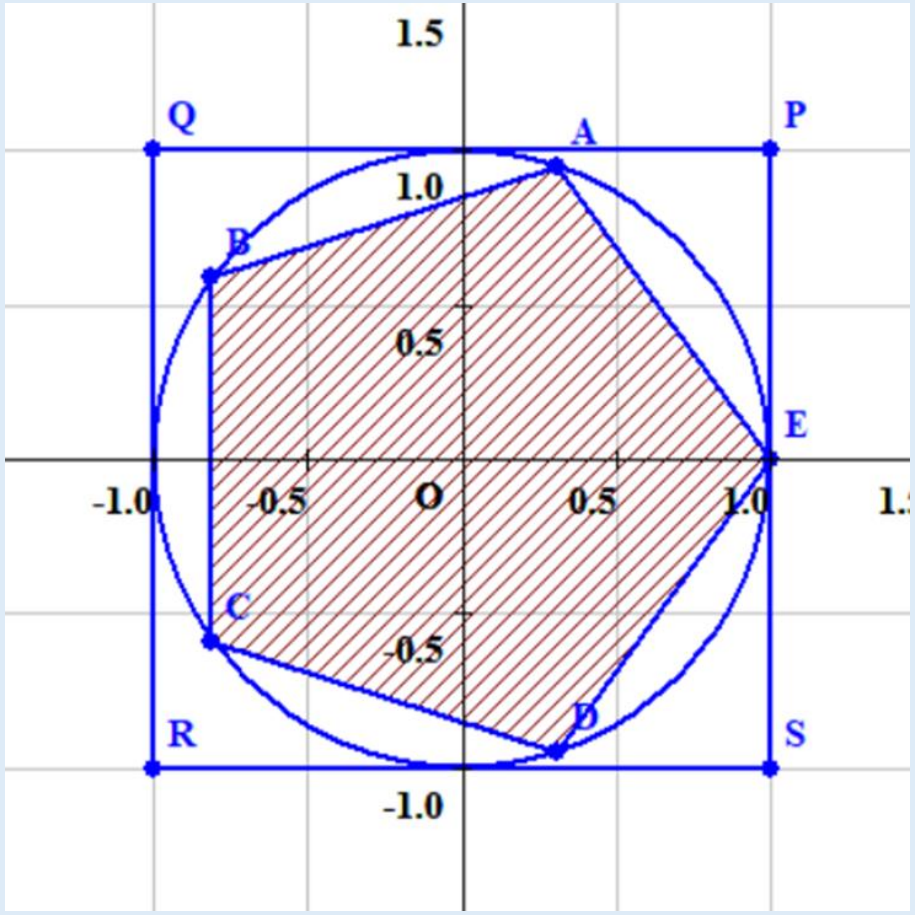
Discussion (generalization)



Discussion (generalization)



Discussion (generalization)



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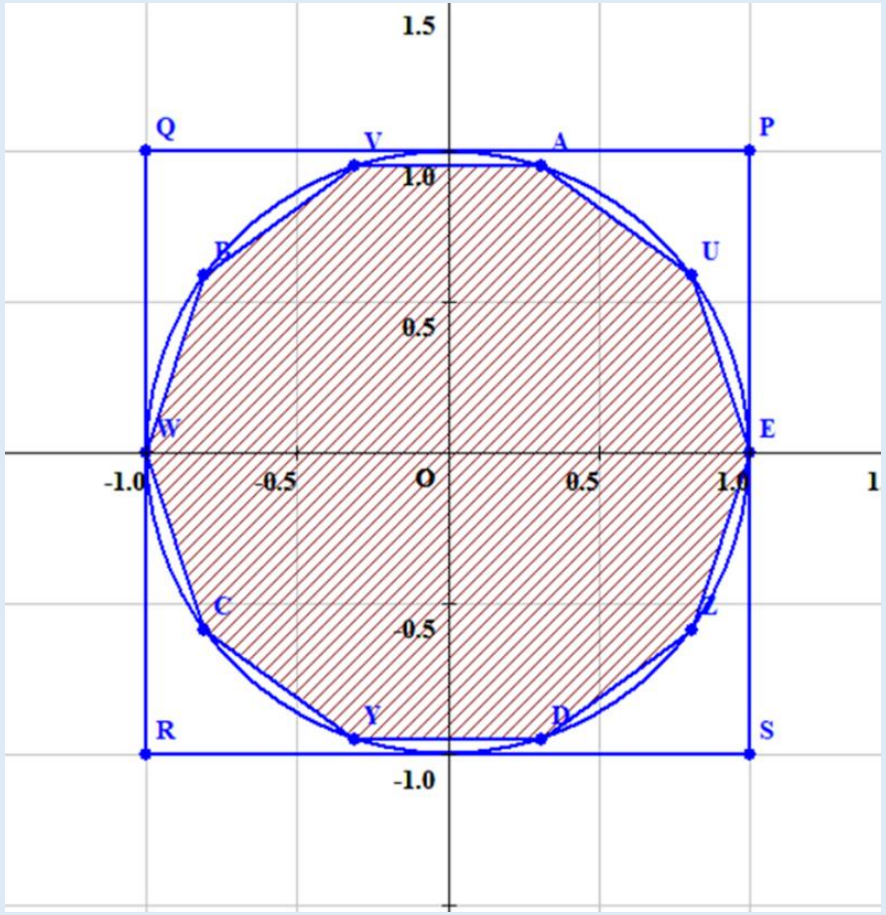

Discussion (generalization)

The lines

$$y = \left(\tan\frac{2\pi}{3p}\right)x \text{ and } y = \left(\tan\frac{4\pi}{3p}\right)x$$

and the regular polygon with p sides

2 steps

The regular polygons with 3p sides

Conclusion

- By using origami, we can—

—Estimate the number of steps required to construct regular polygons.

—Reduce the number of steps required to construct regular polygons.

References

(1)GeoGebra
https://www.geogebra.org/?lang=ja

(2)Jin Nakagawa 2012

(3)フリーイラスト素材集　ジャパクリップ
https://japaclip.com/orizuru/